On the Grayson-Stuhler Filtration of Euclidean Lattices

Renaud Coulangeon, Université de Bordeaux

In this instructional talk I will introduce the notion of semistability for Euclidean lattices, and define the canonical filtration of a Euclidean lattice by semi stable ones. This filtration has many remarkable properties, and has probably not received all the attention it deserves. This theory dates back to Stuhler ([Stu76]) and Grayson ([Gra84]), who used it to build an alternative to Borel Serre’s compactification of locally symmetric spaces.

In short, given a Euclidean lattice \( L \), one can plot in the plane the points \((\dim M, \log \det M)\) as \( M \) varies among all sublattices of \( L \). Their convex hull is bounded below by a certain convex polygon which has two important properties:

- each of its vertices corresponds to a unique sublattice \( M \) of \( L \).
- these sublattices form a chain, which we call the Grayson-Stuhler filtration of \( L \).

Among other noteworthy properties, this filtration is invariant under automorphisms and scalar extension. As time allows, I will also speak about a conjecture regarding the behaviour of this filtration with respect to tensor product.

References

